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SUMMARY

INTRODUCTION
Literature Review
Parisi, Gianfrancesco, Giliberto & Giudici (2015), *Applied Mathematics*
Giudici & Parisi (2015), *XXIV MBF International Rome Conference*

METHODOLOGY
Dynamic Models

EMPirical FINDINGS
Data
Estimated Parameters
Predictive Performance

CONCLUSIONS
Motivation: Monetary Rate Pass Through

Bank interest rates on loans to Corporates up to 1 Mln

BR (%)

France
Germany
Ireland
Italy
Neth.
Port.
Spain
Euribor

Jan 03 Jan 04 Jan 05 Jan 06 Jan 07 Jan 08 Jan 09 Jan 10 Jan 11 Jan 12 Jan 13 Jan 14

time

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Dynamic Hierarchical Models for Monetary Transmission
Literature Review

Methodological Review

- Chong et al. (2006) → Error Correction Model.

\[
\begin{align*}
BR_t &= k + \beta \cdot MR_t + \epsilon_t \\
\Delta BR_t &= \alpha \cdot \Delta MR_t + \delta_1(BR_{t-1} - \beta \cdot MR_{t-1} - k) + \\
&\quad + \delta_2(BR_{t-1} - \beta \cdot MR_{t-1} - k) + u_t,
\end{align*}
\] (1)

- Parisi et al. (2015) → Evolution of the ECM.

\[
BR_t = k + \beta \cdot MR_{t-1} + \gamma \cdot \Delta MR_t + \delta \cdot BR_{t-1} + \epsilon_t;
\] (2)


\[
BR_{j,t}^k = \overline{BR}_j^k + \alpha_1 D^{\text{crisis}}_t + \alpha_2 D^{2008}_t + \alpha_3 MR_{t-1}^{ov} + \alpha_4 (MR_{t-1}^{3m} - MR_{t-1}^{ov}) + \alpha_5 (R_{j,t-1}^{10} - R_{t-1}^{10,\text{swap}}) + \\
+ \alpha_6 Y_{t-1}^k + \alpha_7 BR_{j,t-1}^k + \epsilon_t.
\] (3)
PROPOSED MODEL

- New, one-component model;
- Assumptions:
  - banks decide according to the level of bank rates,
  - bank rates depend on both the level and the variations of monetary rates,
  - level of bank rates depends on its previous level → partial reaction to monetary rates changes;
- Proposed model:
  \[ BR_t = k + \beta \cdot MR_t + \gamma \cdot \Delta MR_t + \delta \cdot BR_{t-1} + \epsilon_t; \]  
  \( (4) \)
- Response variable: level of bank rates;
- Explanatory variables: replace the error correction term of the ECM.
PROPOSED MODEL

- Differentiable:

\[
\frac{dBR}{ds} = \beta \cdot \left[ \frac{dMR}{ds} \right]_{s=t} + \gamma \cdot \left[ \frac{d^2MR}{ds^2} \right]_{s=t} + \gamma \cdot \left[ \frac{dBR}{ds} \right]_{s=t-1} \tag{5}
\]

- Derivative of bank rates depends on:
  - speed of monetary rates,
  - acceleration/deceleration of monetary rates,
  - derivative of bank rates with respect to the previous time.
## ESTIMATES

### Error Correction Model:

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<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td>Coeff.</td>
<td>t</td>
<td>Coeff.</td>
<td>t</td>
<td>Coeff.</td>
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<tr>
<td>$k$</td>
<td>-0.133</td>
<td>-3.426</td>
<td>-0.100</td>
<td>-2.542</td>
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<tr>
<td>$\beta$</td>
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<td>29.741</td>
<td>0.341</td>
<td>29.425</td>
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<td>$\alpha$</td>
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<td>0.0909</td>
<td>4.863</td>
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<td>$\delta_1$</td>
<td>-0.286</td>
<td>-5.028</td>
<td>-0.288</td>
<td>-5.513</td>
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<tr>
<td>$\delta_2$</td>
<td>-0.209</td>
<td>-4.194</td>
<td>-0.220</td>
<td>-4.680</td>
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<tr>
<td>$R^2_{\text{long}}$</td>
<td>0.893</td>
<td>0.880</td>
<td>0.287</td>
<td>0.905</td>
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<td>$R^2_{\text{short}}$</td>
<td>0.443</td>
<td>0.485</td>
<td>0.902</td>
<td>0.449</td>
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</table>

### Proposed Model (stepwise model selection):

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</thead>
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<td>Coeff.</td>
<td>t</td>
<td>Coeff.</td>
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<td>-4.561</td>
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<tr>
<td>$\gamma$</td>
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<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.743</td>
<td>25.695</td>
<td>0.746</td>
<td>30.454</td>
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<td>$R^2$</td>
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<td>0.987</td>
<td>0.974</td>
<td>0.998</td>
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PREDICTIVE PERFORMANCES


- Proxy for goodness of estimations = Root Mean Square Error (RMSE);

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<td>0.016</td>
<td>0.003</td>
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<td>0.065</td>
<td>0.069</td>
<td>0.014</td>
<td>0.018</td>
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</table>
**OBJECTIVES**

**PREVIOUS MODELS**

1. Do not account for time-dependent parameters (time-windows/dummies);
2. Chong et al., Parisi et al.: bivariate relationships;

**PROPOSED MODEL**

1. Dynamic models $\rightarrow$ time-dependent relationships $+$ predictions;
2. Dynamic hierarchical structure $\rightarrow$ two stages:
   - $BR_t \sim f_t(MR_t)$, dynamic seemingly unrelated regression (correlated outputs),
   - $\text{Intercept}_t \sim f_t(X_{1,t}, \ldots, X_{N,t})$, multiple dynamic regression.
DLM and Seemingly Unrelated DLM

- **N** countries, 3 kinds of loans;
  \( i \in W \times V, j \in W, h \in V, \) with \( V = \{1, \ldots, N\}, W = \{1, 2, 3\} \);

\[
\begin{align*}
BR_i^i &= k_i^i + \beta_i^i MR_t + \epsilon_i^i, & \epsilon_i^i &\sim N(0, \sigma_{\epsilon_i}^2) \\
k_i^i &= k_{i-1}^i + \omega_{k,t}^i, & \omega_{k,t}^i &\sim N(0, \sigma_{\omega_{k,i}}^2) \\
\beta_i^i &= \beta_{i-1}^i + \omega_{\beta,t}^i, & \omega_{\beta,t}^i &\sim N(0, \sigma_{\omega_{\beta,i}}^2),
\end{align*}
\]

**ASSUMPTION:** in each country, \( BR_t^j \) similarly react to changes in \( MR_t \)

Seemingly Unrelated Dynamic Regression

\[
\begin{align*}
BR_t^h &= (F_t^T \otimes I_3)\Theta_t^h + \epsilon_t^h, & \epsilon_t^h &\sim N(0, V) \\
\Theta_t^h &= (G \otimes I_3)\Theta_{t-1}^h + w_t^h, & w_t^h &\sim N(0, W),
\end{align*}
\]

\[
\Theta_t^h = (k_{t,1}^1, k_{t,2}^2, k_{t,3}^3, \beta_{t,1}^1, \beta_{t,2}^2, \beta_{t,3}^3)^T,
\]

\[
F_t^T = [1 \quad MR_t], \ G = I_2 \text{ and } W = \text{block-diagonal matrix (} W_k, W_\beta \text{)}.
\]
Hierarchical DLM

- State vector $\Theta^h_t$ captures most of the variability of DLM (Latent variable);
- $\Theta^h_t$ explained through a set of **country-specific regressors**, by a Multiple DLM $\rightarrow$ second stage;

**Dynamic Hierarchical Model**

\[
\begin{align*}
BR^h_t &= (F^T_{BR,t} \otimes I_3')\Theta^h_t + \epsilon^h_t, \quad \epsilon^h_t \sim N(0, V_{BR}) \\
\Theta^h_t &= (F^h_{\theta,t})^T \Gamma^j,h + \nu^j,h, \quad \nu^j,h \sim N(0, V_\theta) \\
\Gamma^j,h_t &= G_t \Gamma^j,h_{t-1} + w^j,h, \quad w^j,h \sim N(0, W),
\end{align*}
\]

\[
F_{BR,t} = \begin{bmatrix} 1 \\ MR_t \end{bmatrix}, \quad F^h_{\theta,t} = \begin{bmatrix} 1 \\ X^h_{1,t} \\ \vdots \\ X^h_{p,t} \end{bmatrix}, \quad \Gamma^j,h_t = \begin{bmatrix} \alpha^j,h \\ \gamma^j,h_{1,t} \\ \vdots \\ \gamma^j,h_{p,t} \end{bmatrix}, \quad G = I_{p+1},
\]

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**Dynamic Hierarchical Models for Monetary Transmission**
PARAMETER ESTIMATION AND PREDICTIONS

1. ESTIMATION: Parameters in $\Theta^h_t$ and $\Gamma^h_j$ estimated through maximum log-likelihood;

2. PREDICTION ($q = 1, \ldots, 12$):

   - Predicted parameters:
     \[
     \begin{align*}
     \hat{\beta}_q &= \hat{\beta}_{q-1} + \hat{\omega}_{\beta,q} \\
     \hat{\alpha}_q &= \hat{\alpha}_{q-1} + \hat{\omega}_{\alpha,q} \\
     \hat{\gamma}_q &= \hat{\gamma}_{q-1} + \hat{\omega}_{\gamma,q}
     \end{align*}
     \]

   - Predicted Monetary Rates:
     \[
     \begin{align*}
     \Delta MR &\sim N(0, \sigma^2) \\
     MR_q &= MR_{q-1} + \Delta MR_q
     \end{align*}
     \]

Predicted State Vector and Bank rates

\[
\begin{align*}
\hat{BR}_q^i &= \hat{k}_q^i + \hat{\beta}_q^i MR_q; \\
\hat{\Theta}_q^i &= \hat{\alpha}_q^i + \sum_{z=1}^{p} \hat{\gamma}_q^i, z \hat{X}_q^h_z
\end{align*}
\]

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Dynamic Hierarchical Models for Monetary Transmission
**Bank Rates on Loans**

*Bank interest rates on loans to Corporates up to 1 Mln*

*Bank interest rates on loans to Corporates over 1 Mln*

*Bank interest rates on loans to Households for Mortgages*
Estimated Parameters

**Seemingly Unrelated Dynamic Regression**

- $k$ - Loans to Corporates $<$ 1 Mln
- $k$ - Loans to Corporates $>$ 1 Mln
- $k$ - Loans to Households for Mortgages

**Estimated Parameters**

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- Dynamic Hierarchical Models for Monetary Transmission
Main findings

1. $\beta_t^i$ almost constant;
2. loans to large corporates much more explained by monetary rates;
3. $k_t^i$ heterogeneous, especially after 2009 and for loans to households $\rightarrow$ country-specific factors;
4. SME: after the crisis, $k_t^i$ differentiate according to core and peripheral countries.
**Dynamic Hierarchical Model - Exogenous Variables**

**Country-specific factors:**

1. Interest rates on 10-years government bonds;
2. Interest rates on deposits (aggregate national level);
3. GDP growth rate.

**Regression Coefficients:**

\[
F_{\theta, t} = \begin{bmatrix}
1 \\
X_{1,t}^h \\
\vdots \\
X_p,t^h \\
\end{bmatrix} = \begin{bmatrix}
1 \\
Bond_t^h \\
Deposit_t^h \\
\Delta GDP_t^h \\
\end{bmatrix}, \quad \Gamma_t^{j,h} = \begin{bmatrix}
\alpha_j^{j,h} \\
\gamma_{1,t}^{j,h} \\
\gamma_{2,t}^{j,h} \\
\gamma_{3,t}^{j,h} \\
\end{bmatrix},
\]

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Dynamic Hierarchical Models for Monetary Transmission
Estimated Parameters

**Dynamic Hierarchical Model - SME**

- Alpha - Loans to Corporates < 1 Mln
- Gamma_1 - Loans to Corporates < 1 Mln
- Gamma_2 - Loans to Corporates < 1 Mln
- Gamma_3 - Loans to Corporates < 1 Mln

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Dynamic Hierarchical Models for Monetary Transmission
Dynamic Hierarchical Model - Large Corp.
Dynamic Hierarchical Models for Monetary Transmission

Estimated Parameters

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Dynamic Hierarchical Models for Monetary Transmission
Dynamic Hierarchical Model - Main Findings

Core countries

1. Germany: dependence only on monetary rates (pivotal country);

France & Ireland

1. France: mixed between core (large corporates) and peripheral countries (SME and households);
2. Ireland: close to peripheral countries in the pre-crisis period, and to core countries in the latest years.

Peripheral countries

1. Italy, Portugal, Spain: strong dependence on exogenous variables for all kinds of loans.
Predicted Bank Rates

Observed and Estimated Rates - France

Observed and Estimated Rates - Germany

Observed and Estimated Rates - Ireland

Observed and Estimated Rates - Italy

Observed and Estimated Rates - Netherlands

Observed and Estimated Rates - Portugal

Observed and Estimated Rates - Spain

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Dynamic Hierarchical Models for Monetary Transmission
## Predictive Performance

### Out-of-sample Tests

<table>
<thead>
<tr>
<th>Country</th>
<th>Loans</th>
<th>$RMSE_1$</th>
<th>$RMSE_2$</th>
<th>$\Delta_{RMSE}%$</th>
<th>$MAPE_1$</th>
<th>$MAPE_2$</th>
<th>$\Delta_{MAPE}%$</th>
</tr>
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<tbody>
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<td>Fra</td>
<td>corp 1</td>
<td>0.0387</td>
<td>0.0155</td>
<td>-60.0%</td>
<td>0.0873</td>
<td>0.0416</td>
<td>-52.4%</td>
</tr>
<tr>
<td></td>
<td>corp 2</td>
<td>0.0552</td>
<td>0.0309</td>
<td>-43.9%</td>
<td>0.1161</td>
<td>0.0813</td>
<td>-30.0%</td>
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<tr>
<td></td>
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<td>0.0835</td>
<td>0.0339</td>
<td>-59.4%</td>
<td>0.0759</td>
<td>0.0437</td>
<td>-42.4%</td>
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<td>Ger</td>
<td>corp 1</td>
<td>0.0529</td>
<td>0.0529</td>
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<td>0.0655</td>
<td>0.0655</td>
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<td>0.1501</td>
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<tr>
<td>Ire</td>
<td>corp 1</td>
<td>0.2271</td>
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<td>Ita</td>
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<tr>
<td>Net</td>
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<td>0.0301</td>
<td>-43.5%</td>
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</table>
CONCLUDING REMARKS

- Good predictive performance of dynamic models (better for hierarchical one);
- Transmission mechanism of MR heterogeneous across time:
  1. Financial crisis: homogeneous reactions across countries;
- Transmission mechanism of MR heterogeneous across loans:
  1. Large corp.: most affected by monetary rates;
  2. SME, households: depend on country-specific factors.
- Transmission mechanism of MR heterogeneous across countries:
  1. Core: most affected by monetary rates;
  2. Peripheral: dependence on bank risk (deposits), sovereign risk (bonds), corporate risk ($\Delta GDP$).

Transmission of Monetary Policy after the crisis

1. Effective in core countries;
2. Problematic in peripheral countries.